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LMI approach for exponential stabilization of continuous-time delayed Takagi-Sugeno systems

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Abstract: This paper deals with the analysis and design of controllers for a class of continuous-time delayed Takagi-Sugeno fuzzy systems. Sufficient conditions for the exponential stability are derived based on a fuzzy Lyapunov function. These conditions are expressed in the form of linear matrix inequalities, that depend on the maximum bound of the time delay and the desired decay rate of the system. A Non parallel distributed compensation controller is considered. The controller design problem is presented as an optimization under linear matrix inequalities constraints. This fuzzy controller provides a maximum domain of constraints determined by the time derivative of the membership functions, while guaranteeing the desired decay rate.The effectiveness of the proposed approaches is shown through a numerical example.

1. INTRODUCTION

Fuzzy systems in the form of the Takagi-Sugeno (T-S) models have attracted much attention in the last decade. These fuzzy dynamic models were proposed by (Takagi and Sugeno, 1985) to represent nonlinear dynamical systems. It has been shown that the T-S models give an effective way to represent complex nonlinear systems by a piecewise interpolation of several linear models through membership functions. These fuzzy models are described by a family of fuzzy IF-THEN rules that represent local linear input-output relations of the system. The overall fuzzy model of the system is achieved by the fuzzy mixing of these local linear models. Thus, controller design methodologies can be developed to achieve stability and performance for the nonlinear system. Therefore, Takagi-Sugeno models are considered a powerful solution to bridge the gap between linear control and fuzzy logic control targeting complex nonlinear systems (Takagi, 1995), (Wang et al., 1996).

The stabilization problem for systems in T-S fuzzy model has been studied extensively and numerous methods have been proposed (Kim et al., 2000) , (Nachidi et al., 2010). For instance, the paper of (Feng, 2006) mainly surveys the stability of fuzzy control based on T-S models, where three types of Lyapunov functions are considered: a common quadratic Lyapunov function, a piecewise quadratic Lyapunov function and a fuzzy Lyapunov function. The control design is carried out based on a fuzzy model via the so-called parallel distributed compensation (PDC) scheme (Wang et al., 1996). The idea is that a linear feedback control is designed for each local linear model, and the resulting overall controller is a fuzzy blending of each individual linear controller, so it is nonlinear in general. The greatest difficulty in using the continuous-time PDC approach with a fuzzy Lyapunov function is that sufficient conditions are usually in the form of bilinear matrix inequalities (BMIs). To address this problem, in Tanaka et al. (2003), a non PDC was further adopted, and results based on this idea were obtained by Nachidi et al. (2009). However, dealing with the stability of T-S systems, few of these papers considered the effect of the time delays that occur in many dynamical systems such as biological systems, chemical systems, nuclear reactions, electrical networks, etc. Nonlinear systems with time delay constitute basic mathematical models of real phenomena. The stability and synthesis of time delay systems is an important issue addressed by many authors and for which surveys can be found in several monographs, for example, Gu (2003) and Kolmanovskii and Richard (1999). In the context of T-S systems with time delay, Cao and Frank (2001) and Guan and Chen (2004) can be mentioned, where delay-independent and delay-dependent stability conditions were obtained. However, the results considered common quadratic Lyapunov functions, which leads to much conservativeness in the stability tests. To avoid this conservatism, fuzzy Lyapunov functions that include the membership functions have recently been used to establish stability and stabilization conditions for T-S systems (Tanaka et al., 2007). Inspired by the work of (Nachidi, 2009) in which stability conditions in LMIs form, for continuous-time fuzzy systems have been derived in the Lyapunov sense without requiring a priori knowledge of bounds on the time-derivative of each membership func-

tion, the main goal of this paper is twofold: first, extend the result of (Nachidi, 2009) to the case of continuous systems with time delay. Second, to design a fuzzy controller for these systems in the form of a non parallel distributed compensation (Non-PDC) scheme. The proposed stability and stabilization approach are represented in terms of optimization problems with LMIs constraints, which can be solved efficiently by using the existing LMIs solvers.
The paper is organized as follows: a description of the Takagi-Sugeno systems with time delay is presented in Section II. In section III, stability conditions are given for autonomous time delay T-S systems using the fuzzy Lyapunov-Krasvoskii approach. Then, using the algorithm developed in the previous section, a fuzzy controller based on a Non-PDC scheme is proposed for the closed loop system. In section IV, a numerical example is presented to demonstrate the effectiveness of the proposed approach. Finally, some conclusions are drawn in section V.
Notation: The notation used throughout the paper is quite standard: $\mathbb{R}^n$ is the n-dimensional real-valued space. For a real-valued matrix P, P^T denotes its transpose, $P > 0$ means that P is symmetric positive-definite, $\lambda_M(P)$ and $\lambda_m(P)$ are maximum and minimum eigenvalues of P, respectively; while $*$ is used to denote the transposed elements in the symmetric positions. $\mathbb{I}, \mathbf{0}$ denote the identity and zero matrices with compatible dimensions, respectively. For any nonsingular square matrix X, X^{-1} is the inverse of X.

2. SYSTEM DESCRIPTION

A T-S model (Takagi and Sugeno, 1985) is described by fuzzy IF-THEN rules. The ith rule of the fuzzy model, with $(i = 1, \dots, L)$, is as follows
Rule i:

$$IF\ z_1(t)\ is\ F_{i1} \text{ and } \dots \text{ and } z_l(t)\ is\ F_{il} \text{ THEN}$$
$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{1i} x(t-\tau) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \tag{1}$$

where τ is an unknown time delay, assumed to be constant and bounded (i.e., $0 \le \tau \le \bar{\tau}$), $x(t) \in \mathbb{R}^n$ denotes the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the output of the system. F_{ij} is the fuzzy set, $A_i \in \mathbb{R}^{n\times n}$, $A_{1i} \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{p\times n}$ are known constant matrices, L is the number of IF-THEN rules, and $z_1(t)$,$z_2(t)$,$\dots$, $z_l(t)$ are the fuzzy variables.
The initial conditions are given by

$$x(t) = \psi(t), \ \forall t \in [-\bar{\tau}, 0]. \tag{2}$$

The overall fuzzy system (Σ) is inferred as follows:

$$(\Sigma) : \dot{x}(t) = \sum_{i=1}^{L} h_i(z(t)) \left[A_i x(t) + A_{1i} x(t-\tau) + B_i u(t)\right].$$

where $h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{L} \mu_i(z(t))}$, $\mu_i(z(t)) = \prod_{j=1}^{l} F_{ij}(z_j(t))$,
with $F_{ij}(z_j(t))$ representing the grade of membership of $z_j(t)$ in F_{ij}.
The normalized membership functions $h_i(z)$ are C^1 functions and satisfy, for all t that

$$h_i(z(t)) \ge 0 \ , \ \text{for}\, i = 1, 2, \dots, L \text{ and } \sum_{i=1}^{L} h_i(z(t)) = 1.$$

The T-S fuzzy model in (1) is a general nonlinear time-varying equation and has been used to model the behaviors of complex nonlinear dynamic systems. In this paper, we investigate the design of state feedback for stabilization of the delayed T-S system. The theoretical results will be established using fuzzy Lyapunov functions, which make them less conservative than those obtained by Cao and Frank (2001) or Guan and Chen (2004).
Next, we recall some basic definitions and lemmas that are useful to obtain our main results.

Definition 2.1. The decay rate of a continuous time system with internal state $x(t)$ is defined as the largest β such that $\lim_{t\to\infty} e^{\beta t} \parallel x(t) \parallel = 0$ holds for all initial conditions.

Remark 2.1. The stability of continuous time systems corresponds to a positive decay rate, i.e: $\beta > 0$.

The following Lyapunov - Krasovskii theorem can be used to prove the asymptotic stability of the above system.

Theorem 2.1. (Kharitonov, 2003) The system (Σ) is asymptotically stable if there exists a bounded quadratic Lyapunov functional $V(x(t))$ such that, for some $\gamma_1 > 0$, it satisfies $V(x(t)) \ge \gamma_1 \parallel \psi(t) \parallel^2$ and its derivative along the system trajectory satisfies $\dot{V}(x(t)) \le -\gamma_1 \parallel \psi(t) \parallel^2$.

Lemma 2.1. (Wang, 1991) For any $z_1, z_2 \in \mathbb{R}^n$ and symmetric positive definite matrix W, we have

$$2z_1^T z_2 \le z_1^T W z_1 + z_2^T W^{-1} z_2.$$

Lemma 2.2. For symmetric matrices $\Omega_{ijr}, i, j, r = 1 \dots,$ L, the following inequality holds

$$\sum_{i=1}^{L}\sum_{j=1}^{L}\sum_{r=1}^{L} \lambda_i\lambda_j\lambda_r\Omega_{ijr} < 0,$$

with $\lambda_i > 0, i = 1, \dots, L$, if there exist matrices $Y_1 = Y_1^T < 0$ and $Y_2 = Y_2^T < 0$ such that

$$\begin{aligned} &\Omega_{iii} \le (L-1)Y_1 \ , \ 1 \le i \le L \\ &\Omega_{iij} + \Omega_{iji} + \Omega_{jii} \le -Y_1 + (L-2)Y_2, \ 1 \le i \ne j \le L \\ &\Omega_{ijr} + \Omega_{irj} + \Omega_{rij} + \Omega_{rji} + \Omega_{jri} + \Omega_{jir} \le -6Y_2, \\ &\qquad\qquad\qquad\qquad 1 \le i < j < r \le L \end{aligned}$$

Proof 2.1. The proof is based on the following equality:

$$\sum_{i,j,r=1}^{L} \lambda_i\lambda_j\lambda_r\Omega_{ijr} = \sum_{i=1}^{L} \lambda_i^3\Omega_{iii} + \sum_{i=1}^{L}\sum_{i\ne j}^{L} \lambda_i^2\lambda_j(\Omega_{iij} + \Omega_{iji} + \Omega_{jii})$$
$$+\sum_{i=1}^{L-2}\sum_{j=i+1}^{L-1}\sum_{r=j+1}^{L} \lambda_i\lambda_j\lambda_r(\Omega_{ijr} + \Omega_{irj} + \Omega_{rij} + \Omega_{rji} + \Omega_{jri} + \Omega_{jir})$$

The details are omitted due to lack of space ■

3. STABILITY ANALYSIS OF TIME-DELAY T-S FUZZY MODELS

This section presents the proposed stability conditions for the autonomous T-S system with time delay. A fuzzy Lyapunov-Krasovskii functional candidate for the T-S fuzzy time-delay system (1) is introduced. Based on

this function, a new stability criterion is derived for autonomous fuzzy time-delay systems. This stability criterion depends on the maximum bound $\bar{\tau}$ and the desired minimum decay rate β.
For convenience of notations, in the sequel, we will denote $h_i(z(t))$ by $h_i(t)$, and

$$A(h) = \sum_{i=1}^{L} h_i(z(t))A_i \,,\; A_1(h) = \sum_{i=1}^{L} h_i(z(t))A_{1i}. \quad (3)$$

Consider the autonomous equivalent of the delayed T-S system (Σ)

$$\dot{x}(t) = A(h)x(t) + A_1(h)x(t-\tau) \quad (4)$$

The following theorem establishes a delay-dependent stability condition for the system (4).

Theorem 3.1. If there exist scalars $\varphi_1 \geq 0, \ldots, \varphi_L \geq 0$, and symmetric matrices $P_1 > 0, \ldots, P_L > 0, Q_1 > 0, \ldots, Q_L > 0, R > 0$, that give a feasible solution of the following Optimization Problem

$$(OP_1)\left\{\begin{array}{l} \underset{\varphi_i,P_i,Q_i,R}{maximize} \sum_{i=1}^{L}\varphi_i \\ (s.t.)\left\{\begin{array}{l}\Gamma_{ii}^k \leq 0,\;\; 1\leq i,k\leq L,\\ \Gamma_{ij}^k+\Gamma_{ji}^k\leq 0, \quad 1\leq i<j,k\leq L,\\ P_iQ_i=\mathbb{I}\,,\; 1\leq i\leq L,\end{array}\right.\end{array}\right. \quad (5)$$

where $\Gamma_{ij}^k = \begin{bmatrix} \frac{1}{L}\left[P_iA_j + A_j^TP_i + 2\beta P_i + R\right] & * & * \\ \frac{1}{L}e^{\beta\bar{\tau}}A_{1j}^TP_i & -\frac{1}{L}R & * \\ -\varphi_k\mathbb{I} & \mathbf{0} & -Q_k \end{bmatrix}$,

such that the condition $\sum_{i=1}^{L}|\dot{h}_i(z(t))|P_i \leq \sum_{i=1}^{L}\varphi_i^2P_i \;,\; \forall t$, is satisfied, then the T-S system (4) is asymptotically stable with minimum decay rate β.

Proof 3.1. To establish the stability condition for the T-S system (4), the following Lyapunov-Krasovskii functional is selected

$$V(x(t)) = x^T(t)P(h)x(t) + \int_{t-\tau}^{t} e^{2\beta(s-t)}x^T(s)Rx(s)ds, (6)$$

with

$$P(h) = \sum_{i=1}^{L} h_i(t)P_i \;,\; P_i > 0, i = 1,\ldots,L, \;\; R > 0, \quad (7)$$

and $\beta > 0$ is the minimum decay-rate as presented in definition 2.1.
The first term of (6) provides a multiple Lyapunov function, which gives less conservative results than those obtained by using a common quadratic functional for all subsystems, whereas the second term gives a weight on the evolution of the states, to ensure a minimum decay rate.
It can be seen that

$$\gamma_1 \parallel x(t) \parallel^2 \leq V(x(t)) \leq \gamma_2 \parallel x(t) \parallel^2, \quad (8)$$

with

$$\gamma_1 = \underset{1\leq i\leq L}{min}(\lambda_m(P_i)), \gamma_2 = \underset{1\leq i\leq L}{max}(\lambda_M(P_i)) + \frac{1}{2\beta}(1-e^{-2\beta\bar{\tau}})\lambda_M(R).$$

Let us consider the following transformations:

$$\xi(t) = e^{\beta t}x(t) \;,\; \zeta(t) = e^{\beta t}y(t). \quad (9)$$

The T-S system (4) is then equivalently expressed by

$$(\Sigma_1) : \dot{\xi}(t) = \bar{A}(h)\xi(t) + \bar{A}_1(h)\xi(t-\tau).$$

where $\bar{A}(h) = \sum_{i=1}^{L} h_i(t)\left(A_i + \beta\mathbb{I}\right) \,,\; \bar{A}_1(h) = e^{\beta\tau}\sum_{i=1}^{L} h_i(t)A_{1i}$.

We consider the L-K functional for (Σ_1) defined by

$$W(\xi(t)) = \xi^T(t)P(h)\xi(t) + \int_{t-\tau}^{t} \xi^T(s)R\xi(s)ds.$$

We can easily verify that the time derivative of $V(x(t))$ along the solutions of system (4) is a function of the time derivative of $W(\xi(t))$ along the solutions of (Σ_1). That is,

$$\dot{V}(x(t)) = e^{-2\beta t}\dot{W}(\xi(t)) - 2\beta e^{-2\beta t}W(\xi(t)). \quad (10)$$

Therefore, the inequality $\dot{W}(\xi(t)) < 0$ implies that $\dot{V}(x(t)) < 0$. Now, let us compute the time derivative of $W(\xi(t))$ for the system (Σ_1):

$$\begin{aligned}\dot{W}(\xi(t)) &= \xi^T(t)\dot{P}(h)\xi(t) + 2\xi^T(t)P(h)\dot{\xi}(t) + \xi^T(t)R\xi(t) - \\ &\quad \xi_\tau^T(t)R\xi_\tau(t) \\ &= \sum_{i=k}^{L}\dot{h}_k(t)\xi^T(t)P_k\xi(t) + \xi^T(t)R\xi(t) - \xi_\tau^T(t)R\xi_\tau(t) + \\ &\quad +2\xi^T(t)\sum_{i=1}^{L}\sum_{j=1}^{L}h_i(t)h_j(t)P_i[\left(A_j+\beta\mathbb{I}\right)\xi(t) + e^{\beta\tau}A_{1j}\xi_\tau(t)].\end{aligned}$$

where, $\xi_\tau(t) = \xi(t-\tau)$. Taking account of the expression

$$\sum_{i=k}^{L}|\dot{h}_k(z(t))|P_k \leq \sum_{i=k}^{L}\varphi_k^2P_k \;,\; \forall t, \quad (11)$$

then, $\dot{W}(\xi(t))$ can be bounded as follows:

$$\dot{W}(\xi(t)) \leq \eta^T(t)\begin{bmatrix}\sum_{k=1}^{L}\varphi_k^2P_k + \Gamma_1(h) & \Gamma_2(h) \\ * & -R\end{bmatrix}\eta(t),$$

with

$$\Gamma_1(h) = \sum_{i,j=1}^{L} h_i(t)h_j(t)[P_i(A_j+\beta\mathbb{I}) + (A_j+\beta\mathbb{I})^TP_i] + R,$$

$$\Gamma_2(h) = e^{\beta\bar{\tau}}\sum_{i,j=1}^{L} h_i(t)h_j(t)P_iA_{1j},$$

$$\eta(t) = \left[\,\xi^T(t)\;\xi^T(t-\tau)\,\right]^T.$$

Using the expression (10), we can write

$$\dot{V}(x(t)) + \beta V(x(t)) \leq e^{-2\beta t}\eta^T(t)\begin{bmatrix}\sum_{k=1}^{L}\varphi_k^2P_k + \Gamma_1(h) & * \\ \Gamma_2^T(h) & -R\end{bmatrix}\eta(t),$$

if the following condition is satisfied:

$$\begin{bmatrix} \varphi_k^2 P_k + \frac{1}{L}\Gamma_1(h) & \frac{1}{L}\Gamma_2(h) \\ * & -\frac{1}{L}R \end{bmatrix} < 0, \; 1 \le k \le L. \quad (12)$$

Thus, we obtain

$$\dot{V}(x(t)) + 2\beta V(x(t)) \le 0$$
$$\Rightarrow V(x(t)) \le e^{-2\beta t} V(\psi) \le \gamma_2 e^{-2\beta t} \parallel \psi \parallel^2,$$

with γ_2 given by (8). Hence, the Lyapunov-Krasovskii stability theorem concludes that if the condition (12) holds, then system (4) is asymptotically stable in the large.

The condition (12) can be difficult to solve due to the mixed products $\varphi_k^2 P_k$. To overcome this problem, we propose to use the cone complementarity formulation.
Let $P_k = Q_k^{-1}$ in (12): by using the Schur complement Lemma, we obtain

$$\begin{bmatrix} \frac{1}{L}\Gamma_1(h) & \frac{1}{L}\Gamma_2(h) & -\varphi_k \mathbb{I} \\ * & -\frac{1}{L}R & \mathbf{0} \\ * & * & -Q_k \end{bmatrix} < 0, \; 1 \le k \le L. \quad (13)$$

By using the convex sum property $\sum_{i=1}^{L} h_i(t) = 1$, the previous inequality can be written as

$$\sum_{i=1}^{L} h_i^2(t)\Gamma_{ii}^k + \sum_{i<j}^{L} h_i(t)h_j(t)\left(\Gamma_{ij}^k + \Gamma_{ji}^k\right) < 0, \quad 1 \le k \le L,$$

with

$$\Gamma_{ij}^k = \begin{bmatrix} \frac{1}{L}\left[P_i\left(A_j + \beta\mathbb{I}\right) + \left(A_j + \beta\mathbb{I}\right)^T P_i + R\right] & * & * \\ \frac{1}{L} e^{\beta\bar{\tau}} A_{1j}^T P_i & -\frac{1}{L}R & * \\ -\varphi_k \mathbb{I} & \mathbf{0} & -Q_k \end{bmatrix} \quad (14)$$

Therefore, the stability condition (13) holds if the following inequalities are satisfied:

$$\Gamma_{ii}^k \le 0 \;, \; \forall i, k \in [1, L],$$
$$\Gamma_{ij}^k + \Gamma_{ji}^k \le 0 \;, \; \forall i, j, k \in [1, L] \text{ with } i < j.$$

The delay-dependent stability conditions (13) are functions of the parameters $\varphi_k, k = 1, \dots, L$. These parameters largely depend on the information of the membership functions and theirs variations. Hence, to get a less conservative stability condition, we maximize the sum of the parameters φ_k: i.e, $\sum_{k=1}^{L} \varphi_k$, which completes the proof. ■

Remark 3.1. It is easy to check that in the absence of delay ($\bar{\tau} = 0$), the set of stability conditions (5) is equivalent to the one obtained in (Nachidi 2009).

To transform the condition $P_i Q_i = \mathbb{I}$, into LMI form, we use the cone complementarity formulation given in (El Ghaoui et al., 1997).

Lemma 3.1. The equality constraint $P_i Q_i = \mathbb{I}$ holds for $i = 1, \dots, L$, if and only if there exist symmetric matrices $P_1 > 0, \dots, P_L > 0, Q_1 > 0, \dots, Q_L > 0$ such that the optimum of the following optimization problem is achievable and equal to $n \times L$:

$$\begin{cases} \underset{P_i, Q_i}{minimize} \sum_{i=1}^{L} Tr(P_i Q_i) \\ (s.t.) \begin{bmatrix} P_i & \mathbb{I} \\ \mathbb{I} & Q_i \end{bmatrix} \ge 0, \quad 1 \le i \le L \end{cases} \quad (15)$$

Taking account of this previous lemma, the following result, which gives a practical solution than the result in theorem 3.1, is proposed.

Proposition 3.1. If there exist scalars $\varphi_1 \ge 0, \dots, \varphi_L \ge 0$, and symmetric matrices $P_1 > 0, \dots, P_L > 0, Q_1 > 0, \dots, Q_L > 0, R > 0$, that give a feasible solution of the following Optimization Problem

$$(OP_2) \begin{cases} \underset{\varphi_i, P_i, Q_i, R}{minimize} \; w \sum_{i=1}^{L} Tr(P_i Q_i) - (1 - w) \sum_{i=1}^{L} \varphi_i \\ (s.t.) \begin{cases} \Gamma_{ii}^k \le 0 \;, \quad 1 \le i, k \le L \\ \Gamma_{ij}^k + \Gamma_{ji}^k \le 0 \;, \quad 1 \le i < j, k \le L \\ \begin{bmatrix} P_i & \mathbb{I} \\ \mathbb{I} & Q_i \end{bmatrix} \ge 0, \quad 1 \le i \le L \end{cases} \end{cases} \quad (16)$$

where Γ_{ij}^k defined by (14), such that the condition (11) is satisfied, then the T-S system (4) is asymptotically stable in the large with the decay rate β.

To solve the optimization problem (OP_2), we use the algorithm developed in (Nachidi et al., 2010).

Algorithm1
With the scalar $0 < w < 1$ and a small tolerance ε fixed:

Step 1: Set $P_i^0 = Q_i^0 = \mathbb{I}$ for $i = 1, \dots, L$.
Step 2: Solve the LMI optimization :
$\underset{\varphi_i, P_i, Q_i, R}{minimize} \; w \sum_{i=1}^{L} Tr(P_i^l Q_i + Q_i^l P_i) - (1 - w) \sum_{i=1}^{L} \varphi_i$ subject to constraints (16).

Step 3: If the following condition $|\sum_{i=1}^{L} Tr(P_i^* Q_i^*) - n \times L| \le \varepsilon$, is satisfied, then stop, otherwise set $P_i^{l+1} \leftarrow P_i^l$, $Q_i^{l+1} \leftarrow Q_i^l$, and repeat from step 2.

Remark 3.2. The parameter w is considered as a tuning parameter which can be adjusted for both algorithm feasibility and to maximize $\sum_{i=1}^{L} \varphi_i$. The proposed approach in this paper employs fewer LMI variables which implies a higher computational efficiency.

3.1 Controller design for time-delay T-S fuzzy systems

The fuzzy controller is designed using a Non Parallel Distributed Compensation (Non-PDC) approach, proposed in (Guerra and Vermeiren, 2004).This control law was developed to achieve more flexibility in designing fuzzy controllers of Takagi Sugeno systems, based on non-quadratic Lyapunov functions. The Non-PDC controller is expressed as a state feedback controller with a time-varying gain:

$$u(t) = \sum_{i=1}^{L} h_i(t) K_i P^{-1}(h)\, x(t), \quad (17)$$

where the matrix $P(h)$ is given by (7). Then, the closed-loop system (Σ_c) is given by

$$\dot{x}(t) = \left(A(h) + B(h)K(h)P^{-1}(h)\right)x(t) + A_1(h)x(t-\tau)$$

with $B(h) = \sum_{i=1}^{L} h_i(t)B_i$, $K(h) = \sum_{j=1}^{L} h_j(t)K_j$, and the matrices $A(h), A_1(h)$ given by (3).

The result for the stabilizing controller design is as follows.

Theorem 3.2. If there exist scalars $\varphi_1 \geq 0, \ldots, \varphi_L \geq 0$, matrices $K_1, \ldots, K_L$, and symmetric matrices $P_1 > 0, \ldots, P_L > 0, Q_1 > 0, \ldots, Q_L > 0, Y_{11} < 0, \ldots, Y_{1L} < 0, Y_{21} < 0, \ldots, Y_{2L} < 0, R > 0$, that give a feasible solution of the following Optimization Problem

$$(OP_3)\begin{cases} \underset{\varphi_k,P_k,Q_k,R,Y_{1k},Y_{2k}}{maximize} \sum_{k=1}^{L} \varphi_k \text{ subject to} \\ \Omega_{iii}^k \leq (L-1)Y_{1k}, \quad 1 \leq i \leq L, 1 \leq k \leq L \\ \Omega_{iij}^k + \Omega_{iji}^k + \Omega_{jii}^k \leq -Y_{1k} + (L-2)Y_{2k}), \\ \qquad 1 \leq i \neq j \leq L \ , 1 \leq k \leq L \\ \Omega_{ijr}^k + \Omega_{irj}^k + \Omega_{rij}^k + \Omega_{rji}^k + \Omega_{jri}^k + \Omega_{jir}^k \leq -6Y_{2k}, \\ \qquad 1 \leq i < j < r \leq L \ , 1 \leq k \leq L, \\ P_i Q_i = \mathbb{I} \ , \quad 1 \leq i \leq L, \end{cases} \tag{18}$$

with $$\Omega_{ijr}^k = \begin{bmatrix} \bar{\Pi}_{ij} & \frac{1}{L}A_{1j}P_i & -\varphi_k \mathbb{I} \\ * & -\frac{1}{L}e^{-2\beta\bar{\tau}}R & \mathbf{0} \\ * & * & -Q_k \end{bmatrix},$$

$\bar{\Pi}_{ij} = \frac{1}{L}\left(A_jP_i + P_iA_j^T + B_jK_r + K_r^TB_j^T + 2\beta P_i + R\right)$. such that the condition (11) is satisfied, then the T-S system (Σ_c) is asymptotically stable.

Proof 3.2. It follows from Lemmas 2.1, 2.2, and the consideration of the following Lyapunov-Krasovskii function candidate:

$$V(x(t)) = x^T(t)P^{-1}(h)x(t) + \\ + \int_{t-\tau}^{t} e^{2\beta(s-t)}x^T(s)P^{-1}(h)RP^{-1}(h)x(s)ds.$$

■

To be able to satisfy the conditions $P_iQ_i = \mathbb{I}$, we apply Lemma 3.1. Thus, the controller gains K_i are obtained by solving the optimization problem presented by the following proposition.

Proposition 3.2. Given the tuning parameter w, if there exist scalars $\varphi_1 \geq 0, \ldots, \varphi_L \geq 0$, matrices $K_1, \ldots, K_L$, and symmetric matrices $P_1 > 0, \ldots, P_L > 0, Q_1 > 0, \ldots, Q_L > 0, Y_{11} > 0, \ldots, Y_{1L} > 0, Y_{21} > 0, \ldots, Y_{2L} > 0, R > 0$, that give a feasible solution of the following Optimization Problem

$$(OP_4)\begin{cases} \underset{\varphi_k,P_k,Q_k,R,Y_{1k},Y_{2k}}{minimize} \ w\sum_{i=1}^{L} Tr(P_iQ_i) - (1-w)\sum_{k=1}^{L}\varphi_k \\ s.t. \ (18) \text{ and } \begin{bmatrix} P_i & \mathbb{I} \\ \mathbb{I} & Q_i \end{bmatrix} > 0 \ , \ 1 \leq i \leq L \end{cases}$$

and condition (11) is satisfied, then the T-S system (Σ) is asymptotically stable with the control law (17).

Remark 3.3. Similar to (OP_2), the optimization problem (OP_4) is solved by applying Algorithm 1. The weight parameter w can be tuned to enlarge the domain defined by $\varphi_1, \ldots, \varphi_L$. The number of LMIs (18) can be reduced by setting $Y_{1k} = Y_1$ and $Y_{2k} = Y_2$, $\forall k$.

Remark 3.4. Proposition 3.2 provides an optimization method with LMIs constraints for the design of stabilizing fuzzy controllers, which is solvable using the LMI Matlab toolbox.

4. NUMERICAL EXAMPLE

In this section, a numerical example is given to show the effectiveness of the proposed approach.

Consider the continuous time delayed T-S system (Σ) with

$$\bar{\tau} = 0.4,$$
$$A_1 = \begin{bmatrix} 1 & -0.5 \\ 1 & 0 \end{bmatrix}, \ A_{11} = \begin{bmatrix} 0 & -0.25 \\ 0.25 & 0 \end{bmatrix}, \ B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1 & 0.5 \\ 1 & -0.5 \end{bmatrix}, \ A_{12} = \begin{bmatrix} 0.85 & 0 \\ 0 & -1 \end{bmatrix}, \ B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

It is possible to verify that the system is unstable for some initial conditions. Selecting $w = 0.43$, and applying Algorithm 1 for a minimum decay rate $\beta = 0.15$, we obtain a feasible solution for stabilization, that corresponds to

$$\varphi_1 = 0.8258, \ \varphi_2 = 1.0714, \ P_1 = \begin{bmatrix} 4.7994 & -1.5092 \\ -1.5092 & 0.6864 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 3.4143 & -0.7113 \\ -0.7113 & 0.4109 \end{bmatrix}, \ Q_1 = \begin{bmatrix} 0.6751 & 1.4844 \\ 1.4844 & 4.7205 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 0.4580 & 0.7928 \\ 0.7928 & 3.8057 \end{bmatrix}, \ R = \begin{bmatrix} 6.6762 & 2.7200 \\ 2.7200 & 1.5133 \end{bmatrix}.$$

The corresponding state feedback gains are given by

$$K_1 = [\,-15.0994 \ -2.6072\,] \ , \ K_2 = [\,-5.4881 \ -0.7786\,].$$

For simulation purposes, we choose the membership functions given by: $h_1(x(t)) = \frac{1}{1+exp(-2x_1(t))}$, $h_2(x(t)) = 1 - h_1(x(t))$.

Fig.1 shows the state trajectories of the closed loop delayed T-S system (Σ_c) with the non parallel compensation controller given by (17), where the initial condition is $x(t) = [\,1 \ 1.5\,]$ for $t \in [-0.4, 0]$. The results obtained using our approach can be easily compared with those obtained using Theorem 2 in (Cao et al., 2001), to see that our approach gives a better performance. The details are omitted due to lack of space, but a simple comparison is shown in Fig.1-2. It is clearly seen from Fig.3 that the system is stabilized, with a degree of exponential decay rate guaranteed by our stabilization conditions: the norm $\| x(t) \|$ is less than $r(t) = 2.5e^{-0.15t}$.

5. CONCLUSION

This paper has studied the stability and stabilization problems for nonlinear delayed systems via T-S fuzzy modeling and control. To analyze the stability and stabilization problems, the Lyapunov-Krasovskii Theorem is applied. Sufficient conditions on the stability and the existence of state feedback are derived, expressed as linear matrix inequalities. These results are dependent on a known maximum bound of the time delay and the minimum decay

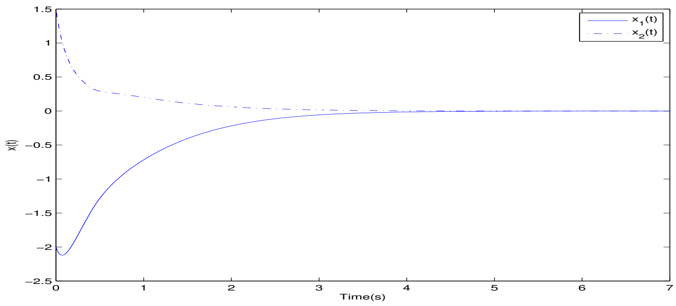

Fig. 1. Evolution of the states of T-S model with proposed non-PDC controller (solid lines), and (dashed lines) PDC controller proposed in Cao et al.(2001).

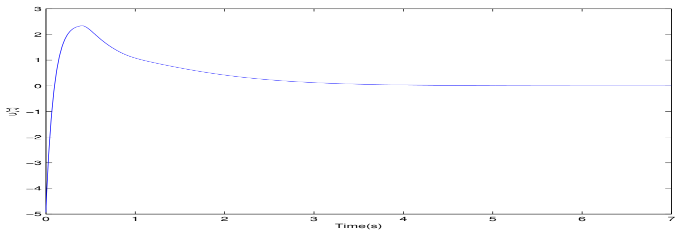

Fig. 2. Evolution of the control input.

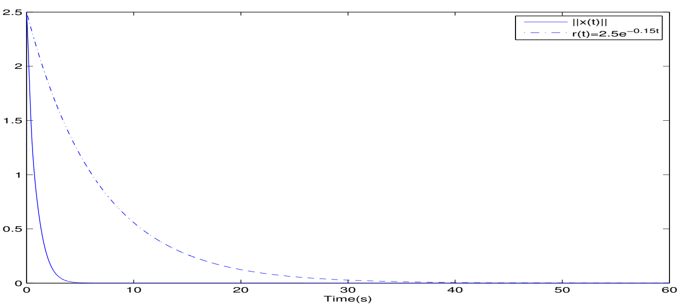

Fig. 3. Evolution of $\| x(t) \|$ of stabilized system.

rate of the system. The advantages of the proposed results are twofold: first, the use of multiple fuzzy Lyapunov-Krasovskii functions reduced the conservativeness; Second, to the best of our knowledge, the obtained results are new, in the sense that the decay rate of T-S models and the time derivation of the membership functions are considered in the state feedback design. As for the prospects of future work, the obtained results will be extended to nonlinear systems with multiple time varying delay. To show the effectiveness, the present results have been illustrated by a numerical example.

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